Journal of Engineering Physics and Thermophysics, Vol. 75, No. 6, 2002

MULTICRITERIAL OPTIMIZATION OF THE OPERATION AND DESIGN PARAMETERS OF ELECTRIC-ARC PLASMATRONS OF LINEAR CIRCUITS

O. V. Zhilinskii, T. V. Laktyushina, and A. N. Laktyushin

UDC 621.387.143

A new method of design synthesis of electric arc heaters by prescribed characteristics is suggested. The method has been applied for multicriterial optimization of the conditions of burning an arc with longitudinal eddy stabilization by an air flow in a plasmatron channel.

Among various electric-arc gas heaters the plasmatrons of linear circuits widely used both in scientific studies and in different branches of technology are the most well-known. They are simple in design and structural embodiment and reliable and convenient in service [1]. Moreover, in such plasmatrons the conditions of arc burning have been investigated most fully, which provides the possibility of their calculation and analysis in designing.

The plasmatrons with gas-eddy stabilization of an arc and a smooth outlet electrode are the most numerous.

To optimize the conditions of arc burning in a plasmatron it is necessary to have a mathematical model, i.e., a set of equations for calculation of its integral characteristics. Using the dependences [12] for generalized current-voltage characteristics (CVC) and thermal characteristics of an air plasmatron and the expressions for arc power and enthalpy of a plasma jet, we will formulate a mathematical model of a one-chamber plasmatron with an outlet electrode constant in diameter:

$$U = 1290 \left(\frac{I^2}{Gd}\right)^{-0.15} \left(\frac{G}{d}\right)^{0.30} (Pd)^{0.25} , \tag{1}$$

$$\frac{1-\eta}{\eta} = 0.585 \cdot 10^{-4} \left(\frac{I^2}{Gd}\right)^{0.27} \left(\frac{G}{d}\right)^{-0.27} (Pd)^{0.30} , \tag{2}$$

$$N = UI , \tag{3}$$

$$\Delta h = UI\eta / G . \tag{4}$$

Expressions (1) and (2) are obtained by generalizing the experimental data on arc stabilization by an air flow and are verified within wide ranges of: current I = (50–5000) A; air-flow rate: $G = (1 \cdot 10^{-3}$–$2.2)$ kg/sec; the outlet electrode diameter: $d = (1$–$7.6) \cdot 10^{-2}$ m; pressure $P = (1$–$100) \cdot 10^5$ N/m^2. As follows from (1)–(4), the characteristic feature of the stated problem on optimization of the conditions of arc burning in a linear plasmatron is the presence of four quantity indices (criteria). Therefore, below we will consider procedures providing solution of this problem in the multicriterial statement.

In general, it is rather rare when the level of perfection and efficiency of a technical object can be evaluated by a single index of quality. Most often it is necessary to have a whole set of criteria, the majority of which are interrelated, since a change in one obligatorily entails a change in another. The existing relations between the indices

---

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 6, pp. 25–28, November–December, 2002. Original article submitted April 25, 2002.

and design limitations on their values necessitates a search for a compromise and a choice of not potentially possible, in principle, extreme values for each index but of such at which the other indices will also have an acceptable value.

The same problems need to be solved by the developers of plasma technology since all the characteristics are interrelated and for each characteristic restrictions exist that depend on the targeted use of a device.

The necessity of conducting design calculations with allowance for many contradictory and interrelated quality indices has led to a search for new specific methods and approaches to solving multicriterial problems.

It should be noted that often researchers try to avoid a multicriterial problem by reducing it to a noncriterial one. In all probability, this can be explained by the fact that currently, for solving problems of optimum designing, which are usually reduced to problems of nonlinear programming, a great number of numerical methods have been developed and a designer has the possibility of choosing the method most suitable for a particular problem.

However, as experience has shown, this leads to a statement and solution of a problem not adequate to the initial one, and in this case none of the optimization methods saves one from obtaining an incorrect result.

In a general form, the problem of taking a solution Z, with several quality indices being present, can be represented by an ordered set of elements (a cortege) [3]:

$$Z = \langle t, \mathbf{S}, \mathbf{K}, \mathbf{F}, f, r \rangle , \tag{5}$$

where t is the statement of the problem, $\mathbf{S}$ is the set of design solutions (alternatives), $\mathbf{K} = \{K_1, ..., K_n\}$ is the set of quality indices (criteria), $\mathbf{F} = \{F_1, ..., F_n\}$ is the set of criteria scales or particular purposes each of which is related to one component of the vector criterion $\mathbf{K}$, f is the mapping of alternatives (versions) into the set of vector estimates in criterial space, and r is the solution rule.

Representation (5) makes it possible to take into consideration such features of a design problem as the multicriterial, multidimensional character of the set of solutions, the multidimensionality of the space of parameters, and the presence of restrictions on the parameters and indices in the form of parametric constraints and requirements for the quality indices of a system.

We will consider the definitions of the elements of model (5) with allowance for the specificity of the plasmatron of a linear current reflected in Eqs. (1)–(4) and the suggested method of solution of a multicriterial problem. The method realizes the basic principle of system designing [4] that necessitates a combination of the formal and logic elements with nonformal ones in the solution procedures. As the formalized element, the method of search based on the use of mathematical sequences possessing the best uniformity characteristics [5] is adopted, while the method of attaining particular goals serves as the nonformalized one [6, 7].

The statement of problem t contains the goal of creating a linear plasmatron with a required level with respect to voltage, power, thermal efficiency, and enthalpy of a plasma jet. The process of design must be terminated by the choice of a single version from a permissible set $\mathbf{S}$ of design solutions that could correspond to the aim of the design formulated in a technical task (TT).

The set $\mathbf{S}$ represents a combination of the possible design solutions separated on a set of the design and operation parameters: I, G, P, and d. Their optimum values must be determined in the course of solving the design problem.

Each solution is evaluated by criteria K_1, ..., K_4, which are the quality indices of the plasmatron: $K_1 = U$, $K_2 = \eta$, $K_3 = N$, and $K_4 = \Delta h$. For each of the criteria a desired range of its values is prescribed, the set of which forms the four-dimensional space of the criteria.

With the aid of the mapping f the set of solutions $\mathbf{S}$ is placed in correspondence with the set of vector estimates $\mathbf{K}$ in the criterial space. The mapping f: $\mathbf{S} \to \mathbf{K}$ is realized in the problem of search for optimum parameters of the plasmatron with the aid of mathematical model (1)–(4).

The solution rule r realizing the system of preferences of the person making a decision (PMD) represents a principle of comparison of the vector estimates. In the present work, the method of successive attainment of particular purposes is used [6, 7], which implies stage-by-stage solution of the problem of multicriterial optimization. In this case, optimization of an object is carried out successively with regard to the importance of a criterion. The solution rule r orders the set of vector estimates $\mathbf{K}$, while the inverse mapping f^{-1} also allows ordering of the set of design solutions $\mathbf{S}$.

In using the method of successive attainment of goals, in each stage the problem of fitting the results of solution for a given criterion with contradictory results for a succeeding one emerges. This procedure is efficiently car-

TABLE 1. Requirements of the Technical Task on the Integral Characteristics of the Plasmatron of a Linear Circuit with Longitudinal Eddy Air Blowing of an Arc

Quality indices (characteristics) of the plasmatron	Ranges of the required values	
	min	max
U	500	1000
$(1-\eta)/\eta$	0	0.75
N	200	400
Δh	3000	5000

TABLE 2. Technical Characteristics of the Versions of the Designed Plasmatron

I		II		III		IV	
Version No.	Δh	Version No.	N	Version No.	$(1-\eta)/\eta$	Version No.	U
16	1633	16	153.6	16	0.3650	11	843.1
8	1642	8	198.8	8	0.3893	3	893.8
2	2264	12	236.0	6	0.4934	5	917.7
4	2303	6	243.3	4	0.5251	6	973.3
12	2494	4	256.8	10	0.5816	15	992.1
14	2536	10	272.1	12	0.6019	13	1039
6	2633	2	298.1	13	0.6296	14	1132
11	2885	14	311.4	11	0.6356	7	1144
13	2931	5	321.2	2	0.6718	1	1166
7	3001	1	349.8	3	0.6918	9	1190
1	3021	3	357.5	1	0.7156	10	1209
9	3136	11	358.3	14	0.7461	12	1348
10	3219	9	386.7	5	0.7517	16	1365
5	3622	13	389.6	9	0.9067	2	1490
3	3757	15	471.2	7	1.034	8	1591
15	4800	7	514.9	15	1.053	4	1712

TABLE 3. Optimum Version of the Air Plasmatron of a Linear Circuit

Operation and design parameters				Technical Data			
I	G	P	d	U	$(1-\eta)/\eta$	N	Δh
400	0.056	$2.5\cdot10^5$	0.022	894	0.69	357.5	3757

ried out with the use of test tables in accordance with [5]. For each criterion a test table is composed in which its values are arranged in increasing order and the numbers of points (for each criterion) of the parameters' probing space are indicated.

Such a representation of the information on the criterion space helps one to easily carry out ordering of the criteria with respect to significance, which is rather popular in multicriterial optimization. This provides the possibility of considerably narrowing the set of optimum versions at the expense of obtaining additional information from a PMD, while the work of the PMD with nonordered criteria often leads to an unsatisfactory result.

In optimization of the plasmatron, the method of successive attainment of particular purposes is employed, which has allowed us to avoid the use of weight coefficients, the function of which in a number of cases turns out to be an insuperable difficulty for the PMD. Therefore, in constructing the nonformalized procedures it is desirable to use questions which can be answered by the PMD without particular difficulties and with sufficient accuracy. In [8], operations concerned with the processing of information which can be sufficiently reliably fulfilled by the PMD are enumerated. The first in this list is the operation of ordering the criteria with respect to significance as well as the operations of determining a satisfactory value of one index and separation of the indices whose values are most unsatisfactory. These are the accuracy and reliability of these operations that determine the correctness of results of the method of successive attainment of particular purposes in optimization of technical objects. At the same time, the availability of computer information on an object (the formalized part of the method) which is complete and ordered with respect to the criteria values substantially facilities the work of a PMD and protects him from losing any of the versions that are of interest to him in choosing the best solution of a design problem.

The procedures of multicriterial optimization described above have been employed in solving the problem of the search for values of design parameters and operating conditions that guarantee a described level of characteristics to a plasmatron.

The plasmatron designed is intended for air heating up to temperatures corresponding to mean-mass enthalpies within the range 2500–5000 kJ/kg. The power range is determined by the required capacity of the setup. It can be reached in different ways; however their choice is restricted by the following fact: at large currents the life of electrodes abruptly decreases. If the current is small, on attaining the required level of power the voltage drop across an arc can turn out to be too great.

In the technical task of creation of a plasmatron, the requirements were put forward for each of the quality indices given in Table 1.

In this case, the space of parameters is a four-dimensional hyperparallelepiped bounded by the limits of change of each of the parameters:

$$I = 100 - 500 \text{ A} , \tag{6}$$

$$G = (4.5 - 9)\cdot 10^{-2} \text{ kg/sec} , \tag{7}$$

$$d = (1 - 2.6)\cdot 10^{-2} \text{ m} , \tag{8}$$

$$P = (1 - 7)\cdot 10^{5} \text{ N/m}^{2} . \tag{9}$$

The volume of the four-dimensional hyperparallelepiped limited by conditions (6)–(9) was probed with the aid of trial points ($L = 16$) whose coordinates (the sets of parameters I, G, d, P) determine the possible versions of a designed object.

By Eqs. (1)–(4), the values of voltage drop on an arc, thermal efficiency, arc power, and enthalpy of an air plasma at the outlet were calculated for each of the sixteen versions. For these indices test tables have been composed in which their values in increasing order and the numbers of the corresponding trial points (Table 2) are given.

The test tables were proposed to a PMD for successive viewing.

The most important quantity for technological applications of the present plasmatron is enthalpy; therefore, Table 2, I, corresponding to the criterion Δh, is the first we have analyzed. Its examination has revealed seven points satisfying the limitations on enthalpy. The next stage is the examination of Table 2, II for arc power N. Five points are found in it that simultaneously satisfy the limitations on enthalpy and power. After the examination of Table 2, III we revealed three versions that satisfy the limitations on enthalpy, power, and thermal efficiency. Finally (Table 2, IV), version No. 3 has been found, for which the requirements of the technical task are fulfilled for all characteristics of the plasmatron.

The values of the design and operation parameters of the optimum version of the electric-arc heater of a linear circuit are given in Table 3.

CONCLUSIONS

1. A solution of the multicriterial problem of the choice of optimum parameters of the plasmatron on the basis of correct successive decrease of the power of the multidimensional set of criteria produced by a computer with the aid of the mathematical model of the plasmatron is suggested.

2. In the formalized part of the solution, provision is made for the possibility of effective and purposeful construction of the sets of parameters and criteria of the required power, the level of which can be varied depending on the quantity of criteria and the chosen ideology of the nonformal part.

3. The procedures of the nonformal part of a solution of the problem of optimization of the plasmatron parameters are formed from the condition of minimization of the power of the criterial set proposed to a PMD which allows the latter to make a more substantiated choice.

4. The described procedures of the multicriterial optimization of the plasmatron of a linear circuit are not related ideologically to a particular object, which allows effective use of this procedure for solving other problems of multicriterial optimization and other technological objects, materials, and technologies.

NOTATION

U, arc voltage, V; η, thermal efficiency; N, arc power, kW; Δh, enthalpy of the plasma jet, kJ/kg; I, arc current, A; G, air-flow rate, kg/sec; d, diameter of the discharge channel, m; P, gas pressure in the channel, N/m^2; L, number of trial points.

REFERENCES

1. M. F. Zhukov, V. Ya. Smolyakov, and B. A. Uryukov, *Electric Arc Heaters* (*Plasmatrons*) [in Russian], Moscow (1973).
2. M. F. Zhukov (ed.), *Principles of Design of Linear Plasmatrons* [in Russian], Novosibirsk (1979).
3. A. N. Danchul and L. Ya. Poluyan, *System-Engineering Problems of Creation of Automatic Design Systems*, in: A. V. Petrov (ed.), *Development of Automatic Design Systems* (in 10 vols.) [in Russian], Vol. 2, Moscow (1990).
4. V. P. Bykov, *Methodical Support of Automatic Design Systems in Machine Building* [in Russian], Leningrad (1989).
5. I. M. Sobol' and R. B. Statnikov, *Formulation of Certain Problems of Optimal Design by Computers* [in Russian], Preprint No. 24 of the Institute of Applied Mathematics of the USSR Academy of Sciences, Moscow (1977).
6. D. I. Batishchev, *Methods of Optimal Design* [in Russian], Moscow (1984).
7. V. L. Volkovich, *Kibernetika,* No. 5, 68–73 (1968).